A demand-and-supply approach to markets, core stability, and mechanism design

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Abstract

This paper uses demand-and-supply diagrams to study a production economy with a numéraire, providing elementary proofs to fundamental economic results. We demonstrate that the competitive outcome lies within the core of the economy's market game and illustrate core convergence to the competitive equilibrium when the economy expands. We further explain the impossibility of efficient trade under private preferences and incentive constraints. Our unified graphical approach makes economic theory more intuitive and easier to teach.

Keywords: invisible hand, demand-and-supply diagram, competitive equilibrium, the core, core convergence, VCG mechanism, impossibility of efficient trades
JEL Classification: A20, C71, D51, D82

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1 Introduction

Demand-and-supply diagrams are indispensable for teaching the theory of competitive markets. They offer a simple yet powerful way to illustrate the interplay of market forces, the notion of competitive equilibrium, and surplus generation from trade. Key dynamics such as the price adjustment process to eliminate excess demand or supply are visually explained without relying on complex mathematics. These graphical tools provide intuitive insights that complement the mathematical sophistication of general equilibrium theory (Arrow and Debreu, 1954; Debreu, 1959; McKenzie, 1959). However, as the teaching of economic theory advances, visual demonstrations often give way to algebraic proofs and formal optimization models. Most of the diagrams familiar to students—such as the Edgeworth box, indifference curves, and production possibility frontiers—are grounded in consumer and producer theories. As a result, students lack visual tools that develop intuition for advanced topics, including coalitional market games and mechanism design.

This paper shows that demand-and-supply diagrams are useful beyond competitive markets, extending to market games and mechanism design. We focus on a production economy with multiple buyers and sellers, where money functions as a *numéraire*. Using standard demand-and-supply diagrams, we can prove graphically three major results in economic theory: (i) the *Core Stability of Competitive Equilibrium*, which establishes the competitive outcome maximizes total surplus from trade and resists coalitional deviations; (ii) the *Edgeworth's Conjecture*, which shows the core shrinks to the competitive equilibrium as the number of participants increases to infinity; and (iii) the *Vickrey's Impossibility Theorem*, which shows that a market maker must inevitably incur a deficit when eliciting true preferences and implementing efficient allocation under private information. Unlocking these graphical tools makes economic theory more accessible and unified for learning and teaching.

To demonstrate the simplicity of our graphical approach, consider the Core Stability of Competitive Equilibrium.[1] This fundamental result shows not only that competitive markets maximize the gains from trade but also that they motivate buyers and sellers to participate in larger, unified markets instead of fragmenting into smaller submarkets—a property known as *core stability*. Due to its mathematical intricacy, this feature of competitive markets is often underappreciated by students.[2] Many only encounter it

[1] This result is among the most profound in economics. Nobel laureate Vernon Smith captured its essence, asking "At the heart of economics is a scientific mystery... as deep and inspiring as the forces that bind matter... How is order produced from freedom of choice?" (Smith, 1982).

[2] While an Edgeworth box visually shows that competitive equilibrium allocations lie within the core of an exchange economy, its restriction to two agents without production makes it less persuasive.

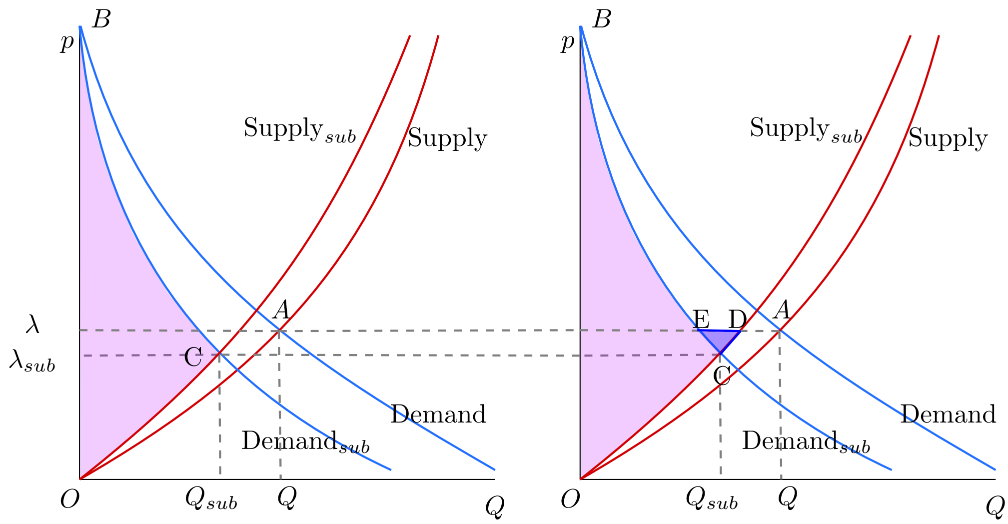

Figure 1: Core Stability Explained: The left shaded triangle shows the surplus from a subgroup trading internally, while the right shows the greater surplus from trading within the full market.

after taking advanced theory courses. Figure 1 provides a clear visual explanation of core stability, making it accessible to students with a basic understanding of microeconomics. In both panels, the market demand and supply intersect at the competitive equilibrium point A, with market-clearing price λ. The area (OAB) illustrates the maximum gains from trade for the whole market achievable under competitive equilibrium. Consider now a submarket consisting of a subset of buyers and sellers, characterized by reduced demand Demand$_{sub}$ and reduced supply Supply$_{sub}$. Within this submarket, the trade surplus is illustrated by the shaded area (OCB), where C, the intersection of its demand and supply curves, determines the submarket's competitive price λ_{sub}. In autarky, when a group trades exclusively among its own members, its surplus is less than the surplus $(ODEB)$ attainable within the integrated market. Integration generates an additional surplus, illustrated by the positive area (CDE), irrespective of whether the submarket price λ_{sub} lies below or above the overall market price λ. Because autarky prevents interaction with external participants, any submarket gains by joining the larger, integrated market.

Related Literature

The core theory in both exchange and production economies is well established. While the stability of the price system is widely accepted by students as a given fact, it has yet to be effectively illustrated using a supply-and-demand diagram. Edgeworth (1881) suggested that core allocations shrink to competitive equilibria as economies expand, a conjecture later proven by Debreu and Scarf (1963).

The foundations of market games and coalitional bargaining were developed by Shapley and Shubik (1969) and further advanced by Aumann and Shapley (1974). Their pioneering work formally connects cooperative game theory with economic equilibrium concepts, providing deep insights into coalition formation, core stability, and surplus distribution. Our graphical approach complements these classical contributions by visually clarifying the relationships among competitive equilibria, market games, and the core, thus enhancing pedagogical effectiveness.

In efficient mechanism design, the Vickrey-Clark-Groves (VCG) mechanism stands out as the revenue-optimal scheme to maximize surplus in settings characterized by private preferences (Vickrey, 1961; Clarke, 1971; Groves, 1973). Vickrey (1961) initially demonstrate the impossibility of efficient trade. In the VCG mechanism, which implements efficient allocations in dominant strategies, the market maker inevitably incurs a deficit. Myerson and Satterthwaite (1983) later generalize this result to a broader class of mechanisms that achieve efficiency in Bayesian-Nash equilibrium, showing no advantage over dominant-strategy equilibrium. The deficit represents the information rent needed to prevent people from strategically misrepresenting their private information. Our analysis of the VCG mechanism and its impossibility result, viewed through the lens of market games, complements the approach of Le (2025).

2 Production economy

We consider a *production economy* with n buyers and m sellers, collected in the sets N and M, respectively. There is a perfectly divisible commodity available for trade.[3] The buyers and sellers have quasilinear preferences represented by their respective value and cost functions. Buyer i enjoys the payoff of $v_i(x_i) - p_i$, when consuming x_i and paying p_i. Seller j earns the profit of $p_j - c_j(y_j)$, when producing y_j and earning p_j. Money serves as the *numéraire*, allowing payoffs and profits to be measured in monetary terms.

We assume that each buyer $i \in N$ has a value function $v_i : [0, a_i] \to \mathbb{R}_+$ that is strictly increasing, strictly concave, and continuously differentiable, with boundary conditions $v_i(0) = 0, \quad v_i'(0) = \omega_B > 0$, and $v_i'(a_i) = 0$. Here, a_i denotes buyer i's maximum demand, and ω_B is the common upper bound of marginal value across all buyers.[4] Under these assumptions, the consumer problem $L_i(\lambda) = \max_{x \in \mathbb{R}_+} \{v_i(x) - \lambda x\}$ has a unique solution $D_i(\lambda)$ for every price $\lambda \in [0, \omega_B]$, characterized by the first-order condition $v_i'(D_i(\lambda)) = \lambda$. Each seller $j \in M$ has a cost function, $c_j : [0, b_j] \to \mathbb{R}_+$

[3] The setup can handle multiple commodities, but for simplicity, we focus on a single commodity.

[4] When both a_i and ω_B are infinite, the value function satisfies the Inada conditions (Inada, 1963).

that is strictly increasing, strictly convex and continuously differentiable, with boundary conditions $c_j(0) = 0, c_j'(0) = 0$ and $c_j'(b_j) = \omega_S > 0$. Here, b_j represents the maximum supply capacity of seller j and ω_S is the common upper bound of marginal cost across all sellers.[5] This ensures the producer problem $K_j(\lambda) = \max_{y\in\mathbb{R}_+}\{\lambda y - c_j(y)\}$ has a unique solution $S_j(\lambda)$ for each price $\lambda \in [0, \omega_S]$, given by the first-order condition $c_j'(S_j(\lambda)) = \lambda$. It is convenient to define the price domain as $[0, \omega]$, where $\omega = \max\{\omega_B, \omega_S\}$. Along the price interval, we extend $D_i(p) = 0$ for $p \in [\omega_B, \omega]$ and $S_j(p) = b_j$ for $p \in [\omega_S, \omega]$. For simplicity, we assume a single tradable commodity with common price supports $[0, \omega_B]$ for buyers and $[0, \omega_S]$ for sellers, though both assumptions can be relaxed.

This environment can be viewed from three distinct perspectives: the competitive market where prices drive decisions, a market game focusing on dividing the surplus gained from trade, and efficient mechanism design aimed at eliciting private information and thus achieving optimal allocations.

Competitive market

In this setting, a prevailing price λ dictates self-interested decisions. Each buyer i chooses consumption x_i to maximize his utility $v_i(x_i) - \lambda x_i$ and each seller j selects production y_j to maximize her utility $\lambda y_j - c_j(y_j)$. The optimal consumption and production in response to price are given by the individual demand $D_i(\lambda)$ and individual supply $S_j(\lambda)$. Market equilibrium is reached at the price λ^{CE} where total demand equals total supply:

$$D(\lambda^{CE}) = \sum_{i\in N} D_i(\lambda^{CE}) = \sum_{j\in M} S_j(\lambda^{CE}) = S(\lambda^{CE}).$$

Competitive consumption and production are given by $x_i^{CE} = D_i(\lambda^{CE})$ and $y_j^{CE} = S_j(\lambda^{CE})$ for each buyer i and seller j. Under boundary conditions, market demand decreases strictly from $A_N = \sum_{i\in N} a_i$ to 0 over the price interval $[0, \omega_B]$, while market supply increases strictly from 0 to $B_M = \sum_{j\in M} b_j$ over the price interval $[0, \omega_S]$. These properties guarantee the existence and uniqueness of competitive equilibrium price.[6]

Figure 2 demonstrates how the market achieves equilibrium at point A, through the interaction of demand and supply, establishing a unique equilibrium price λ^{CE}. The market-clearing price optimally coordinates economic activities, maximizing the trade

[5] When both b_j and ω_S are infinite, the cost function satisfies the reverse Inada conditions.

[6] Formally, existence follows from the intermediate value theorem: since $D(0) - S(0) = A_N > 0$, $D(\omega) - S(\omega) = -B_M < 0$ and the excess demand function $D(\lambda) - S(\lambda)$ is continuous on $[0, \omega]$, there must exist a price $\lambda^{CE} \in [0, \omega]$ at which excess demand vanishes. Uniqueness follows from the strict monotonicity of the excess demand function, which results from the strict concavity of the value functions and the strict convexity of the cost functions.

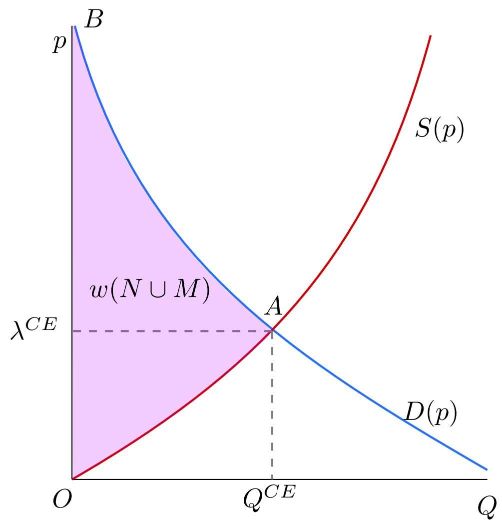

Figure 2: Market Equilibrium - The Intersection of Demand and Supply

surplus, which is represented by the shaded area (OAB).[7]

Market game

The bargaining over trade surplus between buyers and sellers can be modeled as a market game $(N \cup M, w)$. The surplus function w assigns to each coalition $C_B \cup C_S$ comprising buyers C_B and sellers C_S the maximum gain from trade $w(C_B \cup C_S)$ defined as:

$$\begin{aligned} \underset{(x_i)_{i \in C_B}, (y_j)_{j \in C_S}}{\text{maximize}} \quad & \sum_{i \in C_B} v_i(x_i) - \sum_{j \in C_S} c_j(y_j) \\ \text{subject to} \quad & \sum_{i \in C_B} x_i = \sum_{j \in C_S} y_j. \end{aligned}$$

Inada's conditions guarantee that non-trivial coalitions with at least one buyer and one seller have a well-defined, unique solution based on first-order conditions, while trivial coalitions composed solely of buyers or solely of sellers generate no surplus.

The payoff profile $((u_i)_{i \in N}, (u_j)_{j \in M})$ is in the core of the market game $(N \cup M, w)$ if it satisfies:

$$\text{Feasibility}: \quad \sum_{i \in N} u_i + \sum_{j \in M} u_j = w(N \cup M), \tag{1}$$

$$\text{Coalitional Rationality}: \quad \sum_{i \in C_B} u_i + \sum_{j \in C_S} u_j \geq w(C_B \cup C_S) \quad \text{for all } C_B, C_S. \tag{2}$$

[7] The notation $(P_1, P_2, \dots, P_n)$ represents the area enclosed by the points $P_1, P_2, \dots, P_n$, traced along the corresponding curves.

Figure 2 shows the gain from trade for the grand coalition, $w(N \cup M)$, represented by the area (OAB) below the market demand curve and above the market supply curve. The surplus from trade for a smaller coalition (e.g. $C_B \cup C_S$) can be viewed as the region enclosed by the demand of buyers C_B and the supply of sellers C_S.

Efficient mechanism design

Each buyer and seller knows their own values and costs, but this information is private. The market maker asks them to report their demand or supply curves, which are then used to decide who gets what and who pays or receives how much. However, agents might not be truthful and could exaggerate or downplay their real preferences. To implement efficient allocation under private information, the VCG mechanism depends on reported preferences and imposes externality-based charges to incentivize truthful reporting. In particular, each buyer i pays

$$p_i^{VCG}(\hat{\boldsymbol{v}}, \hat{\boldsymbol{c}}) = \hat{w}(N \cup M \setminus i) - (\hat{w}(N \cup M) - \hat{v}_i(\hat{x}_i^{CE})),$$

and each seller j earns

$$p_j^{VCG}(\hat{\boldsymbol{v}}, \hat{\boldsymbol{c}}) = (\hat{w}(N \cup M) + \hat{c}_j(\hat{y}_j^{CE})) - \hat{w}(N \cup M \setminus j).$$

Here $(\hat{\boldsymbol{v}}, \hat{\boldsymbol{c}}) = ((\hat{v}_i)_{i \in N}, (\hat{c}_j)_{j \in M})$ denotes the reported value and cost functions, $\hat{x}_i^{CE}$ and $\hat{y}_j^{CE}$ are the competitive consumption and production levels derived from the reported preference. The function $\hat{w}(C_B \cup C_S)$ denotes the reported worth of the coalition $C_B \cup C_S$. The VCG mechanism makes truthful reporting a weakly dominant strategy, enabling the market maker to implement an efficient outcome when preferences are reported truthfully. The market maker runs an *ex post deficit* whenever $R = \sum_{i \in N} p_i^{VCG} - \sum_{j \in M} p_j^{VCG} < 0$, across all admissible reported value and cost functions.

3 A unified graphical approach

We first show the competitive equilibrium maximizes surplus from trade by solving the social planner's problem, with the Lagrange multiplier being the competitive price. Using demand-supply diagrams, we confirm that the competitive outcome lies in the core of the economy's market game. Next, in replica economies, we show equal treatment within the core and prove that the core converges to the competitive equilibrium as the market grows, since net gains from adding participants approach zero. Finally, we establish the impossibility of efficient trade under private preferences using the VCG mechanism.

3.1 Core stability of competitive equilibrium

The core stability of competitive equilibrium is one of the most profound ideas in economics. It explains why order emerges from freedom of economic choice. Larger, connected markets work better because no group can break away and trade more efficiently on their own. It is the same reason why farmers' markets hum with activity while roadside stands struggle as more buyers and sellers mean better matches and smoother trades. This principle underpins the success of stock exchanges and online platforms, where size and connectivity drive efficiency. It is Adam Smith's (1776) *invisible hand* in action. Self-interested choice leads to coordination without central control, showing policymakers why thicker markets fuel growth and prosperity.

To explain why the competitive outcome maximizes gains from trade, known as the First Welfare Theorem, we consider the planner's problem of finding a feasible allocation that maximizes the trade surplus. This leads to solving the following convex optimization problem whose solution yields the maximum achievable surplus:

$$\begin{aligned} w(N \cup M) &= \max_{(x_i),(y_j)} \left\{ \sum_{i \in N} v_i(x_i) - \sum_{j \in M} c_j(y_j) \right\} \\ \text{subject to} & \quad \sum_{i \in N} x_i = \sum_{j \in M} y_j \quad \longleftarrow \lambda. \end{aligned} \tag{3}$$

The first order conditions for problem (3) require that the marginal value $v_i'(x_i)$ for each buyer i, and the marginal cost $c_j'(y_j)$ for each seller j, must equal the Lagrange multiplier λ of the consumption-equal-production constraint. By expressing x_i and y_j as functions of λ—yielding individual demand $D_i(\lambda)$ and individual supply $S_j(\lambda)$—and determining λ through the consumption-equal-production constraint, this approach mirrors the process of establishing the market clearing price from market demand and supply. Hence, competitive allocations maximize surplus, attaining $w(N \cup M)$.

To explain the core stability of competitive equilibrium, we model the production economy as a market game $(N \cup M, w)$. Consider the *competitive payoff profile* $(u_k^{CE})_{k \in N \cup M}$, where $u_i^{CE} = L_i(\lambda^{CE})$ for each buyer i, and $u_j^{CE} = K_j(\lambda^{CE})$ for each seller j. To establish core stability, we must verify that this payoff profile satisfies both Feasibility (1) and Coalitional Rationality (2).

The competitive payoff profile satisfies Feasibility (1) as the total exactly matches the grand coalition's trade surplus $w(N \cup M)$, given market clearing condition $\sum_i x_i^{CE} =$

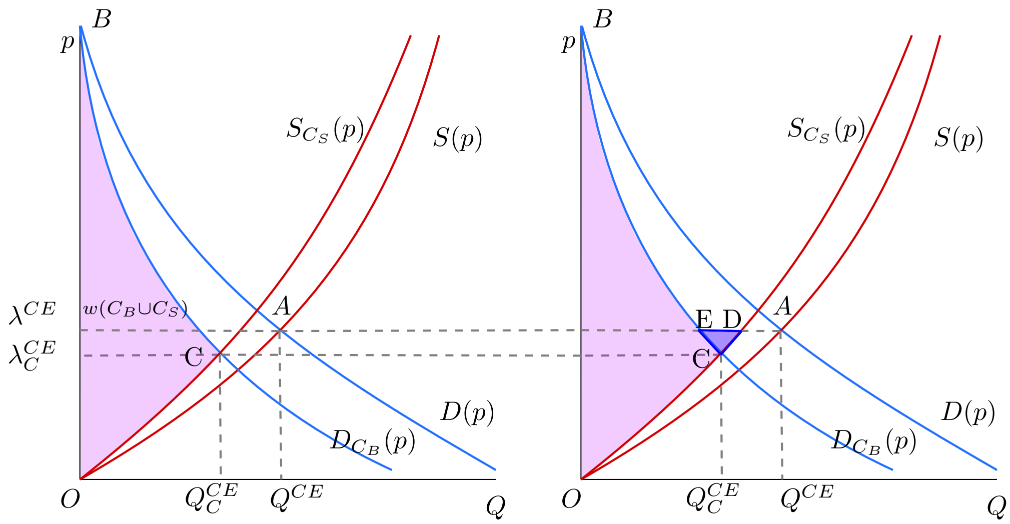

Figure 3: Graphical proof of Coalitional Rationality

$\sum_j y_j^{CE}$. Figure 3 visually confirms Coalitional Rationality (2). For any given coalition, say $C = C_B \cup C_S$, the supply curves $S_{C_S}(p)$ and the demand curves $D_{C_B}(p)$ are confined to buyers C_B and sellers C_S, respectively. The shaded area labeled (OCB) in the left panel quantifies the autarky trade surplus $w(C_B \cup C_S)$, which is less than the shaded area labeled $(ODEB)$ in the right panel, representing the aggregate competitive payoffs of the coalition. The difference (CED) measures the additional gains from trade achieved by buyers C_B and sellers C_S through interacting with others outside the coalition.[8] If $N \setminus C_B$ and $M \setminus C_S$ each contain at least one agent, the relative position of λ^{CE}_C and λ^{CE} is ambiguous. Nonetheless, the surplus difference (ECD) is always non-negative.

A simple numerical example with linear demand and supply can clarify core stability:

Example 1 (Core Stability of Linear Economy)**.** There are n buyers, each with demand $D_i(p) = 1 - p$, and m sellers, each with supply $S_j(p) = p$, defined for $p \in [0, 1]$. This corresponds to value function $v_i(x) = x - \frac{x^2}{2}$ and cost function $c_j(y) = \frac{y^2}{2}$, each defined for quantities in $[0, 1]$. The market demand is $D(p) = n(1 - p)$ and the market supply is $S(p) = mp$, resulting in competitive price $\lambda^{CE} = \frac{n}{m+n}$. The grand coalition's surplus is $w(N \cup M) = \frac{mn}{2(m+n)}$. For any coalition $C_B \cup C_S$ of n_0 buyers and m_0 sellers, its autarky surplus is:

$$w(C_B \cup C_S) = \frac{m_0 n_0}{2(m_0 + n_0)},$$

[8] Formally, this inequality is proven from the fact that the competitive price λ^{CE} minimizes the total indirect utilities and profits, given by $W(\lambda) = \sum_{i \in N} L_i(\lambda) + \sum_{j \in M} K_j(\lambda)$. The minimum value of this function is exactly $w(N \cup M)$. The proof uses Hotelling's Lemma $L_i'(\lambda) = -D_i(\lambda)$, Shephard's Lemma $K_j'(\lambda) = S_j(\lambda)$, and the first-order condition $W'(\lambda^{CE}) = 0$. Observant readers may notice this also underpins *Walras' tâtonnement*, where price adjusts gradually in proportion to the excess demand, causing $W(\lambda)$ to decrease until the price reaches λ^{CE}.

and its surplus when trading with the rest of the market is:

$$\overline{w}(C_B \cup C_S) = n_0 \times \frac{m^2}{2(m+n)^2} + m_0 \times \frac{n^2}{2(m+n)^2}.$$

Thus, the core stability of competitive equilibrium follows from coalitional rationality:

$$\overline{w}(C_B \cup C_S) - w(C_B \cup C_S) = \frac{(n_0 m - m_0 n)^2}{2(m_0 + n_0)(m+n)^2} \geq 0. \quad (4)$$

Second Welfare Theorem

In production economies, the lack of budget constraints and personal endowments limits the role of redistribution. In contrast, exchange economies bring redistribution to the forefront. Consider an *exchange economy* consisting of n agents, collected by N. Each agent $i \in N$ possesses an initial endowment $e_i \geq 0$ and a strictly increasing, strictly concave, and differentiable value function $v_i(x_i)$. The aggregate endowment is given by $E = \sum_{i=1}^n e_i$. Each agent seeks to maximize their utility $v_i(x_i) - \lambda x_i$, which generates individual demand functions $D_i(\lambda)$. Competitive equilibrium is attained at the price λ^{CE}, where the sum of individual demands equals the total available endowment. This competitive equilibrium allocation maximizes total surplus, formally described by:

$$w(N) = \max \left\{ \sum_{i \in N} v_i(x_i) \right\} \quad \text{subject to} \quad \sum_{i \in N} x_i = \sum_{i \in N} e_i.$$

In equilibrium, agents whose equilibrium demands surpass their initial endowments act as buyers, whereas those whose demands fall below their endowments become sellers. The Core Stability Theorem parallels the setup of production economies.

In this exchange context, the Second Welfare Theorem takes on a narrower but more powerful interpretation: in the absence of budget constraints (implying an infinite budget endowment), the competitive equilibrium remains unaffected by any redistribution of initial endowments, provided the aggregate endowment remains constant. Consequently, any Pareto-optimal payoff profile $(u_i)_{i \in N}$ (that is, $\sum_{i \in N} u_i = w(N)$) satisfying $u_i \geq v_i(x_i^{CE}) - \lambda^{CE} x_i^{CE}$ for every $i \in N$ can be realized through suitable reallocations of initial endowments. Agents who initially hold no endowment achieve the lower-bound utility of $v_i(x_i^{CE}) - \lambda^{CE} x_i^{CE}$. This observation has important policy implications: redistribution is not a tool to expand the economic pie, but a means to divide it more equitably. By shifting initial endowments among agents without changing the total resources, policymakers can reduce payoff inequality while preserving economic efficiency.

3.2 Core convergence to competitive equilibrium

Core convergence to competitive equilibrium, known as the Edgeworth's conjecture, illustrates that competitive prices equitably solve surplus distribution as markets expand. Larger markets naturally balance supply and demand, preventing any subgroup from benefiting by trading separately. Policymakers thus gain insight into the importance of promoting competition for scalable efficiency and stability.

The key simplification lies in establishing the principle of *equal treatment of equals* for every core-stable payoff profile of replica economies. If identical agents receive unequal payoffs, the disadvantaged agents can form a coalition to block this inequality. As the economy is replicated infinitely, the core shrinks, ultimately converging to the competitive equilibrium. This convergence occurs because the additional surplus contributed by adding an individual agent (excluding their own direct contribution) diminishes to zero as the market grows.

Formally, we replicate the original market r times, creating an r-fold economy with rn buyers and rm sellers, denoted as $N^{(r)}$ and $M^{(r)}$, respectively. This forms the r-fold market game $(N^{(r)} \cup M^{(r)}, w)$. The primary goal is to analyze how the core evolves as the market expands with increasing r.

First, equal treatment is necessary within the core of a replica economy because total surplus scales proportionally with replication: $w(N^{(r)} \cup M^{(r)}) = rw(N \cup M)$. If agents of the same type, meaning those with identical preferences, receive different payoffs, the least advantaged among them could form a blocking coalition C of n buyers and m sellers.[9] Since this coalition mirrors the original market of $N \cup M$, it should be able to generate the same surplus as the original market. Any deviation from equal treatment would imply that this coalition could block the allocation, violating Coalitional Rationality (2). Therefore, all agents of the same type must be treated equally. This principle allows for a convenient characterization of the core using payoff profiles from the original market, rather than specifying payoffs from the entire r-fold market.

Second, we examine which payoff profiles from the original market survive the core induced by repeated market replication. From the Welfare Theorem, competitive payoff profile remains in the core of $(N^{(r)} \cup M^{(r)}, w)$ for any r, as the market clearing price for any replica economy stays the same $\lambda^{CE,(r)} = \lambda^{CE}$. However, proving that the competitive payoff profile is the only survivor is more challenging. This hinges on the observation that the marginal gain from trade by adding an extra buyer or seller becomes negligible

[9] For example, when the market is duplicated ($r = 2$), if buyer i_1 receives a lower payoff than her twin i_2, then i_1 would join a blocking coalition. Even if i_1 and i_2 are treated equally, one of them would still join the coalition if a different identical pair is treated unequally.

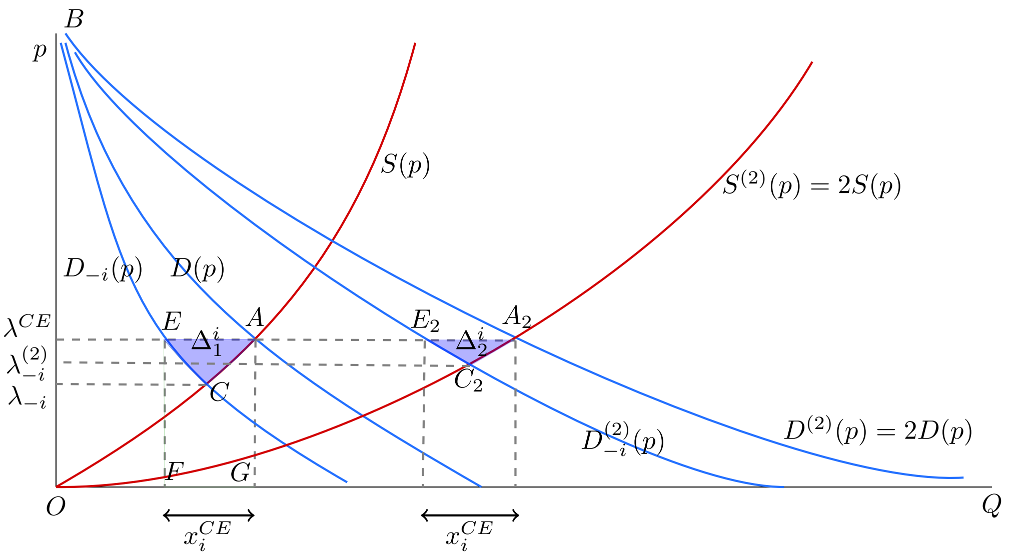

Figure 4: Graphical illustration of surplus gain Δ_r^i approaching zero—a key step in proving core convergence to the competitive equilibrium

in a sufficiently large market. To demonstrate this, define $\Delta_r^{i_0}$ as the surplus gain of all participants excluding buyer i_0 from trading with i_0 under the competitive equilibrium of an r-fold economy:

$$\Delta_r^{i_0} = w(N^{(r)} \cup M^{(r)}) - u_{i_0}^{CE} - w(N^{(r)} \cup M^{(r)} \setminus i_0).$$

Similarly, let $\Delta_r^{j_0}$ be the surplus gain of all participants excluding seller j_0 from trading with j_0; formally, $\Delta_r^{j_0} = w(N^{(r)} \cup M^{(r)}) - u_{j_0}^{CE} - w(N^{(r)} \cup M^{(r)} \setminus j_0)$.

Figure 4 demonstrates the convergence of Δ_r^i to 0. Originally, the market operates with demand $D(p)$ and supply $S(p)$. Upon market duplication, both demand and supply shift to the right, becoming $D^{(2)}(p)$ and $S^{(2)}(p)$, effectively doubling their initial quantities. Despite this expansion, the efficient consumption level for buyer i remains unchanged, as the market clearing price λ^{CE} stays constant (thus, $AE = A_2E_2$ remains). However, the impact of buyer i's absence on the competitive price is reduced by half, $\lambda^{CE} - \lambda_{-i}^{(2)} \approx \frac{1}{2}(\lambda^{CE} - \lambda_{-i})$. This is due to the doubling of the slopes of the demand and supply curves, which lessens buyer i's influence on price. The curved triangle (AEC) represents Δ_1^i and the curved triangle $(A_2E_2C_2)$ represents Δ_2^i. Since the bases AE and A_2E_2 remain constant at x_i^{CE} and the price effect is halved, the area of $(A_2E_2C_2)$ is approximately half of (AEC), confirming $\Delta_r^i \to 0$ at a convergence rate of $1/r$.[10] A similar argument applies to the convergence of Δ_r^j for any seller j.

[10] A rigorous proof requires showing that $\lambda_{-i_0}^{(r)} \to \lambda^{CE}$ as $r \to \infty$, along with applying a second-order Taylor expansion to the surplus gap. Specifically, we express $\Delta_r^{i_9} =$

Third, suppose to the contrary that a payoff profile, say $((u_i^C)_{i\in N},(u_j^C)_{j\in M})$, other than the competitive payoff profile $((u_i^{CE})_{i\in N},(u_j^{CE})_{j\in M})$ survives the core of r-fold replica economy. Then there must exist at least one agent—either a buyer i_0 for whom $u_{i_0}^C > u_{i_0}^{CE}$, or a seller j_0 for whom $u_{j_0}^C > u_{j_0}^{CE}$. Without loss of generality, assume the strict inequality holds for buyer i_0. To reach a contradiction, consider the surplus gain $\Delta_r^{i_0}$. By the coalitional rationality of $((u_i^C)_{i\in N},(u_j^C)_{j\in M})$, for every r we have:

$$w(N^{(r)}\cup M^{(r)}) - u_{i_0}^C \geq w(N^{(r)}\cup M^{(r)}\setminus i_0).$$

Rewriting the inequality in terms of $\Delta_r^{i_0}$ yields $\Delta_r^{i_0} \geq u_{i_0}^C - u_{i_0}^{CE}$. Since $u_{i_0}^C - u_{i_0}^{CE} > 0$ by assumption, the surplus gain is bounded below by a fixed positive number regardless of r. This contradicts the earlier result that $\Delta_r^{i_0} \to 0$ as $r\to\infty$. Hence, no payoff profile other than the competitive one can survive in the core of replica economies.

To illustrate the convergence of $\Delta_r^{i_0}$ and the core shrinkage, we present the following simple numerical example, which builds on the setup in Example 1:

Example 2 (Core Convergence in a Linear Economy)**.** Consider again the linear production economy with n buyers, each with demand $D_i(p) = 1-p$, and m sellers, each with supply $S_j(p) = p$, defined over the price interval $[0,1]$. Replicating this economy r times does not affect the competitive price, which remains $\lambda^{CE} = \frac{n}{n+m}$. The convergence of $\Delta_r^{i_0}$ to zero, along with its convergence rate, can be verified using Equation 4, which yields the following expression:

$$\Delta_r^{i_0} = \frac{m^2}{2r(m+n)^2(m+n-1/r)}.$$

In the case of bilateral trade, where $(n=m=1)$, core shrinkage can be characterized precisely. Any payoff pair (u_B,u_S) that lies in the core of the r-fold bilateral production economy must satisfy $u_B+u_S = \frac{1}{4}$ and for any $r_B, r_S \leq r$, coalitional rationality holds:

$$r_B u_B + r_S u_S \geq \frac{r_B r_S}{2(r_B+r_S)}.$$

When $r_B = r$ and $r_S = r-1$, it follows that $u_B \geq \frac{r-1}{8r-4}$. Similarly, if $r_B = r-1$ and $r_S = r$, we obtain $u_S \geq \frac{r-1}{8r-1}$. Combined with the constraint $u_B + u_S = \frac{1}{4}$, these inequalities imply that the only payoff pair that survives in the core under repeated replication of the bilateral economy is the competitive pair $\left(\frac{1}{8},\frac{1}{8}\right)$.

$\left[\sum_{i\in N} rL_i(\lambda) + \sum_{j\in M} rL_j(\lambda) - L_{i_0}(\lambda)\right]_{\lambda=\lambda_{-i_0}^{(r)}}^{\lambda^{CE}}.$

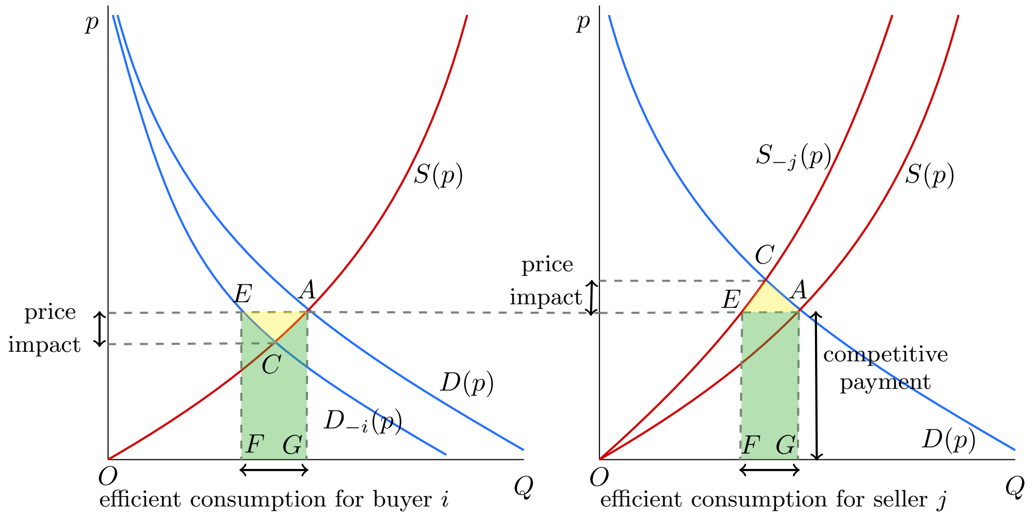

Figure 5: Comparison of VCG payments versus the competitive payments

3.3 Impossibility of efficient trade

Imagine a market maker trying to organize trade efficiently without knowing what buyers and sellers truly want or value. Unlike Adam Smith's (1776) *invisible hand*, which assumes prices naturally adjust to balance supply and demand, this market maker plays the role of a *visible hand*. Think of a real estate auctioneer unaware of each bidder's true willingness to pay or a government setting up carbon credit markets without knowing companies' exact emissions costs.

The information gap creates a tension between efficiency—allocating resources to those who value them most—and balancing the budget. According to the Coase Theorem, private bargaining can achieve efficiency when transaction costs are absent and information is perfect. However, in reality, private information and strategic misrepresentation complicate this process. To explore this tension visually, we use demand-and-supply diagrams to contrast competitive equilibrium with the VCG mechanism's dominant strategy. This approach shows why private information acts as a significant transaction cost, violating the Coase Theorem's assumptions.

The VCG mechanism, designed to achieve efficient allocations, charges buyers and compensates sellers based on the externalities they impose. However, it inevitably fails to balance the budget because under VCG, buyers always pay less than their competitive payments, and sellers always earn more than their competitive transfers. This imbalance creates a budget deficit, as the competitive equilibrium balances the budget through its decentralized structure. This is the impossibility of efficient trade, a foundational theorem in mechanism design (Vickrey, 1961; Myerson and Satterthwaite, 1983).

Figure 5 illustrates the impossibility result using two demand-supply diagrams. Both diagrams show demand and supply curves, $D(p)$ and $S(p)$, intersecting at point A—the competitive equilibrium. When buyer i is removed, demand shifts left to $D_{-i}(p)$ by the amount buyer i demands, $D_i(p)$. Similarly, when seller j is removed, supply shifts left to $S_{-j}(p)$ by the amount seller j supplies, $S_j(p)$. The new equilibrium points C reflect the market adjustments in their absence. In the left diagram, buyer i's VCG payment equals their competitive payment ($AGFE$) minus a discount (ACE). In the right diagram, seller j's VCG transfer equals their competitive transfer ($AGFE$) plus a subsidy (ACE). These curved triangles reflect the *information rents*—the extra gains buyers and sellers secure due to private information and the ability to misrepresent preferences strategically. The size of these rents depends primarily on the competitive quantity (the base) and the individual's influence on price (the height).

Under the VCG mechanism, each buyer i pays no more than their competitive payment $\lambda^{CE} x_i^{CE}$, and each seller j earns at least their competitive transfer $\lambda^{CE} y_j^{CE}$. These bounds follow from coalitional rationality, which compares the total surplus with and without each participant at the core-stable competitive payoff. For any buyer i, excluding them from the market leaves the surplus $w(N \cup M \setminus i)$. By the core stability of the competitive equilibrium, this must be less than the surplus of the grand coalition, $w(N \cup M)$, reduced by buyer i's competitive payoff u_i^{CE}. Formally for buyer i,

$$w(N \cup M) - (v_i(x_i^{CE}) - \lambda^{CE} x_i^{CE}) > w(N \cup M \setminus i).$$

Similarly, for a seller j,

$$w(N \cup M) - (\lambda^{CE} y_j^{CE} - c_j(y_j^{CE})) > w(N \cup M \setminus j).$$

These inequalities are strict because removing a participant changes the market-clearing price. Rearranging them shows that buyers pay less under VCG ($p_i^{VCG} < \lambda^{CE} x_i^{CE}$) and sellers receive more ($p_j^{VCG} > \lambda^{CE} y_j^{CE}$) relative to the competitive equilibrium, reflecting discounts and subsidies driven by private information and strategic behavior.

The following example provides a clear numerical illustration of VCG payments and transfers in a linear economy:

Example 3 (VCG Mechanism in a Linear Economy)**.** Consider a slightly more general production economy with n buyers, each with demand $D_i(p) = a_i(1 - p)$, and m sellers, each with supply $S_j(p) = b_j p$, defined over the price interval $[0, 1]$.[11] Buyers and sellers have privately known types a_i and b_j, respectively, drawn from distributions F and

[11] This corresponds to value function $v_i(x) = x - \frac{x^2}{2a_i}$ for $x_i \in [0, a_i]$ and cost function $c_j(y) = \frac{y^2}{2b_j}$ for $y_j \in [0, b_j]$.

G. They report types $\hat{\mathbf{a}} = (\hat{a}_i)_{i\in N}$ and $\hat{\mathbf{b}} = (b_j)_{j\in M}$, which may differ from their true types. Denote $A = \sum_{i=1}^{n} a_i$, $B = \sum_{j=1}^{m} b_j$ and $\hat{A} = \sum_{i=1}^{n} \hat{a}_i$ and their reported sums $\hat{B} = \sum_{j=1}^{m} \hat{b}_j$. The efficient allocations are derived based on the reported competitive price, given by $\hat{\lambda}^{CE} = \frac{\hat{A}}{\hat{A}+\hat{B}}$. Thus, the allocation rules for each buyer i and each seller j are $x_i(\hat{\mathbf{a}}, \hat{\mathbf{b}}) = \frac{\hat{a}_i\hat{B}}{\hat{A}+\hat{B}}$, $y_j(\hat{\mathbf{a}}, \hat{\mathbf{b}}) = \frac{\hat{b}_j\hat{A}}{\hat{A}+\hat{B}}$. VCG payment for buyer i and transfer for seller j are:

$$\begin{aligned} p_i^{VCG}(\hat{\mathbf{a}}, \hat{\mathbf{b}}) &= \frac{\hat{a}_i\hat{A}\hat{B}}{\hat{A}+\hat{B}} - \frac{\hat{a}_i^2\hat{B}^2}{2(\hat{A}+\hat{B})^2(\hat{A}+\hat{B}-\hat{a}_i)}, \\ p_j^{VCG}(\hat{\mathbf{a}}, \hat{\mathbf{b}}) &= \frac{\hat{b}_j\hat{A}^2}{\hat{A}+\hat{B}} + \frac{\hat{b}_j^2\hat{A}^2}{2(\hat{A}+\hat{B})^2(\hat{A}+\hat{B}-\hat{b}_j)}. \end{aligned}$$

The impossibility of efficient trade is confirmed from the following deficit:

$$\sum_{i=1}^{n} \frac{\hat{a}_i^2\hat{B}^2}{2(\hat{A}+\hat{B})^2(\hat{A}+\hat{B}-\hat{a}_i)} + \sum_{j=1}^{m} \frac{\hat{b}_j^2\hat{A}^2}{2(\hat{A}+\hat{B})^2(\hat{A}+\hat{B}-\hat{b}_j)} > 0.$$

4 Conclusion

This paper introduces a clear and intuitive graphical approach using demand-and-supply diagrams to explain three of the most important ideas in economics. First, it shows why economic order naturally emerges from freedom of choice—the stability of competitive equilibrium ensures that no group can trade more efficiently by breaking away. Second, it demonstrates how effectively market outcomes scale, showing that as markets expand, stable and efficient results converge toward competitive equilibrium. Finally, it underscores the difficulties faced by a central market maker in achieving efficient outcomes under private information, emphasizing the inherent tradeoff between efficiency and balanced revenue. By visually simplifying these fundamental ideas, the framework makes key economic concepts accessible to students and non-specialists without excessive reliance on complex mathematics.

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